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Institute for Algebra and Geometry
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26	27	28	29	30	$\sum$

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Student Nr.:

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Worksheet No.6
Advanced Mathematics I

Exercise 26:

The function $f : \mathbb{R}\backslash\{-2\} \to \mathbb{R}$ is defined as

$$f(x) = \begin{cases} \dfrac{5x \cdot |x-3|}{x^2 - x - 6} & , \quad x \in \mathbb{R}\backslash\{-2, 1, 3\} \ , \\ y_1 & , \quad x = 1 \ , \\ y_2 & , \quad x = 3 \ . \end{cases}$$

Is it possible for f to be continuous at $x = 1$ and $x = 3$ with a suitable choice of y_1, y_2? Give the appropriate values, or show that none exist.

Exercise 27: Let the set of complex numbers D be defined as $D := \{z \in \mathbb{C} : |z| \leq 1 \text{ und } \operatorname{Im}(z) \geq 0\}$. Consider the function $f : D \to \mathbb{R}$,

$$f(z) = \left|\operatorname{Im}\left((2-i)z\right)\right| .$$

Are there points in D, where f takes its maximum and minimum? If so, find them and compute the corresponding function values.

Exercise 28: Show that for any positive constants a, b, c the equation

$$\frac{(a+b)x + a - b}{x^2 - 1} + \frac{c}{x-2} = 1$$

has solutions in the intervals $[-1, 1]$ and $[1, 2]$, respectively.

Exercise 29: Find the number of solutions of the equation

$$2x^5 - 6x^3 + 2x = 4x^4 - 6x^2 + 1$$

in the interval $I = [-2, 2]$ and justify your answer.

Exercise 30:

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous 2π-periodic function (i.e. $f(x) = f(x + 2\pi)$ for all $x \in \mathbb{R}$). We use f to define the function $h : \mathbb{R} \to \mathbb{R}$ by

$$h(x) = f\big(x\big) + f\big(x + \pi\big) - f\big(x + \tfrac{1}{2}\pi\big) - f\big(x + \tfrac{3}{2}\pi\big).$$

Show that h has a zero in the interval $\left[0, \frac{\pi}{2}\right]$.

(b) Let the topography of a terrain be given by a continuous function $F : \mathbb{C} \to \mathbb{R}$, where $F(x + iy)$ is the height of the terrain at the point (x, y). A quadratic table is placed in this terrain such that its center is located at the origin of the coordinate system. The legs of this table form a square[1] with the corners $a, b, c, d \in \mathbb{C}$. Let (r, φ) be the polar coordinates of a. The table is stable if and only if

$$\frac{F(a) + F(c)}{2} = \frac{F(b) + F(d)}{2}$$

holds[2]. Show that the table can be turned to a position where it is stable.

Due date: Please hand in your homework until Thursday, December 9, 12:00 into the AM1/2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

[1] approximately if the height-differences of the terrain are not too large

[2] This can be seen by the following argument: Let A, B, C, D be the points in the terrain at a, b, c, d. The table is stable iff these points are contained in a plane. This is the case iff the diagonals AC and BD intersect. For this the heights of the midpoints of AC and BD have to coincide.

Tutorial 6
Advanced Mathematics 1

Exercise T16: Is it possible in each case to choose values y_1, y_2 such that the given functions $f, g : \mathbb{R} \to \mathbb{R}$ become continuous? Calculate those values or prove that a solution is not possible.

$$\text{(a)} \quad f(x) = \begin{cases} \dfrac{x^4 - 10x^2 + 9}{x^2 - 4x + 3} & \text{für } x \in \mathbb{R}\backslash\{1,3\} \\ y_1 & \text{für } x = 1 \\ y_2 & \text{für } x = 3 \end{cases} \qquad \text{(b)} \quad g(x) = \begin{cases} \dfrac{x^3 + 4x^2 + x - 6}{x^3 - 3x + 2} & \text{für } x \in \mathbb{R} \setminus \{1, -2\} \\ y_1 & \text{für } x = 1 \\ y_2 & \text{für } x = -2 \end{cases}$$

Exercise T17: Show (using the intermediate value theorem) that the following equations are true at least for two $x \in \mathbb{R}$ without explicitly solving the quadratic equations:

$$\text{(a)} \quad x + 1 = 4 - x^2, \qquad \text{(b)} \quad x + 1 = 4x^2 - 24x + 36.$$

Exercise T18: Consider the set $M = \{z \in \mathbb{C} : |z| \leq 2\}$ and the function $f : \mathbb{C} \to \mathbb{R}$, where

$$f(z) = \operatorname{Re}\left((3 + 4i)z\right) .$$

(a) Decide whether M is open, closed, or compact, respectively.

(b) Determine the maximum and minimum values of f on M.

For detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Wednesday, December 1, 2010, 14:00-15:30.