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Wavelets
Winter Semester 2013/2014

Problem Sheet 4 of November 18, 2013

Exercise 12:
Let $\psi = \sum_{k\in\mathbb{Z}} a_k \chi_{[k,k+1[}$ with $\sum_{k\in\mathbb{Z}} |k||a_k| < \infty$ and $\sum_{k\in\mathbb{Z}} a_k = 0$.

(a) Show that ψ is a wavelet.
Hint: Consider Lemma 2.5 with $\beta = 1$.

(b) Which additional restrictions on $\{a_k\}_{k\in\mathbb{Z}}$ guarantee order 1, 2 and 3?

Exercise 13:
Let ψ be a wavelet of order $N \in \mathbb{N}$ with compact support $[T_1, T_2]$. Define

$$\varrho(x) = \frac{1}{(N-1)!} \int_{T_1}^{x} (x-z)^{N-1} \psi(z)\, \mathrm{d}z\,.$$

Show that $\operatorname{supp} \varrho = [T_1, T_2]$ and

$$\frac{\mathrm{d}^N}{\mathrm{d}x^N} \varrho = \psi\,.$$

These excersises are discussed in the problem class on **Thursday, November 21, 2013**.