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Mathematical Methods in Quantum Mechanics II

11th Exercise Sheet

Exercise 32:
Show that the functional

$$P_1^\alpha(\psi) = \|\nabla\psi\|_2^2 - C\alpha \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \frac{|\psi(x)|^2|\psi(y)|^2}{|x-y|}\, dxdy$$

where C, α are constants is well defined on $H^1(\mathbb{R}^3)$ and bounded from below.

Exercise 33:
Let

$$\mathcal{E}_1^\alpha = \inf_{\psi\in H^1, \|\psi\|_2=1} P_1^\alpha(\psi)$$

where P_1^α is as in Exercise 32. Show that $\mathcal{E}_1^\alpha = \alpha^2\mathcal{E}_1^1$.

Exercise 34:
Let

$$\mathcal{E}_n^{U,\alpha} = \inf_{\|\psi\|_2=1} P_n^{U,\alpha}(\psi)$$

where $P_n^{U,\alpha}$ is as in HW sheet 10 Exercise 30. For $v > 0$ show that $\mathcal{E}_n^{v\alpha,\alpha} = \alpha^2\mathcal{E}_n^{v,1}$.

Exercise 35:
Let

$$f(k) = \sqrt{\alpha}\,\frac{\hat{\rho}(k)}{|k|}$$

where α and ρ are as in HW sheet 10 Exercise 30. Show that for

$$\eta = \sum_{n=0}^{\infty} \frac{f^{\otimes n}}{\sqrt{n!}}$$

there exists a constant c such that $a(k)\eta = cf(k)\eta$.