Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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Karlsruhe, Jan 17, 2019

Name: ....

Exercise Sheet No. 12
Advanced Mathematics I

Exercise 56: Compute the following limits:

$$\text{(a)}\quad \lim_{x\to 0}\left(\frac{1}{x^3+ax^2+x}-\frac{1}{\sin x}\right),\qquad \text{(b)}\quad \lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x},\qquad \text{(c)}\quad \lim_{x\to 0}(\cos x)^{1/x^2}.$$

The coefficient $a\in\mathbb{R}$ in part (a) is a constant.

Exercise 57:

Determine the constant $c\in\mathbb{R}$ such that the function $f(x)=\begin{cases} c, & x=1,\\ \frac{2^{\ln x}-x}{\ln x}, & x\neq 1,\end{cases}$ on $\mathbb{R}_{>0}$ is continuous.

Exercise 58: Calculate all the derivatives $f^{(n)}$, $n=0,1,2,\dots$ of the function f and give the Taylor series for f with center of expansion $x_0=0$. Where does the series in part (a) converge?

$$\text{(a)}\quad f(x)=\cosh\frac{x}{2},\quad x\in\mathbb{R},\qquad\qquad \text{(b)}\quad f(x)=\sqrt{1+x},\quad |x|\le 1.$$

Hint for (b): The derivatives of f have the following form: $f^{(k)}(x)=-(-1)^k\frac{(2k-2)!}{2^{2k-1}(k-1)!}(1+x)^{-\frac{2k-1}{2}}$.

Exercise 59:

(a) Determine the Taylor formula for $m=2$ about the point $x_0=8$ for the function $f(x)=x^{2/3}$, $x\ge 1$, find an expression for the Lagrange form of the remainder.

(b) For natural numbers n and real x, $1+x>0$, show using the Taylor formula that

$$(1+x)^n\ge 1+nx.$$

Exercise 60:

(a) Define the function $f\colon\mathbb{R}\to\mathbb{R}$ by $f(x)=1$ for $x\ge 0$ and $f(x)=-1$ for $x<0$. Show that the function $F(x)=\int_0^x f(t)dt$ is not a primitive (antiderivative) of f.

(b) Given $g\colon\mathbb{R}_{>0}\to\mathbb{R}$ defined by $g(x)=\int_{2\pi}^x \frac{\sin(t)}{t}dt$. Find the Taylor polynomial $p_2(x)$ of degree 2 about $x_0=2\pi$.

Due date: Your written solutions are due at 14:00 on Tuesday, **29 January, 2019**. Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

http://www.math.kit.edu/iag6/edu/am12018w/en