Connectivity of arc complexes - Part II

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Next week we will continue to study different kinds of arc complexes and show that they are highly connected.

Connectivity of joins

Let K and L be two simplicial complexes with vertices K_0 and L_0 , respectively. The join $K \star L$ of K and L is the simplicial complex with vertices $K_0 \sqcup L_0$ and

$$K \star L = \left\{ \sigma = \langle a_0, \dots, a_k, b_0, \dots, b_l \rangle \mid \langle a_0, \dots, a_k \rangle \in K \text{ and } \\ \langle b_0, \dots, b_l \rangle \in L \right\},\$$

i.e. it is the smallest complex which contains for each simplex $\sigma_1 = \langle a_0, \ldots, a_k \rangle$ in K and each simplex $\langle b_0, \ldots, b_l \rangle$ in L the simplex $\langle a_0, \ldots, a_k, b_0, \ldots, b_l \rangle$.

Suppose that K is contractible. What can you say about $K \star L$? More generally, what can you say about the connectivity of $K \star L$, if K is c_1 -connected and L is c_2 -connected?

A special arc complex

Let S be a surface of genus 1 with one boundary component and p a point on the boundary, $\Delta = \{p\}$. Consider the arc complex $A(S, \Delta)$. How does it look like? Do you recognise it from other occasions as e.g. elementary number theory or elementary geometry?